

UNIVERSITY OF WATERLOO  
FACULTY OF ENGINEERING  
Department of Electrical &  
Computer Engineering

ECE 204 *Numerical methods*

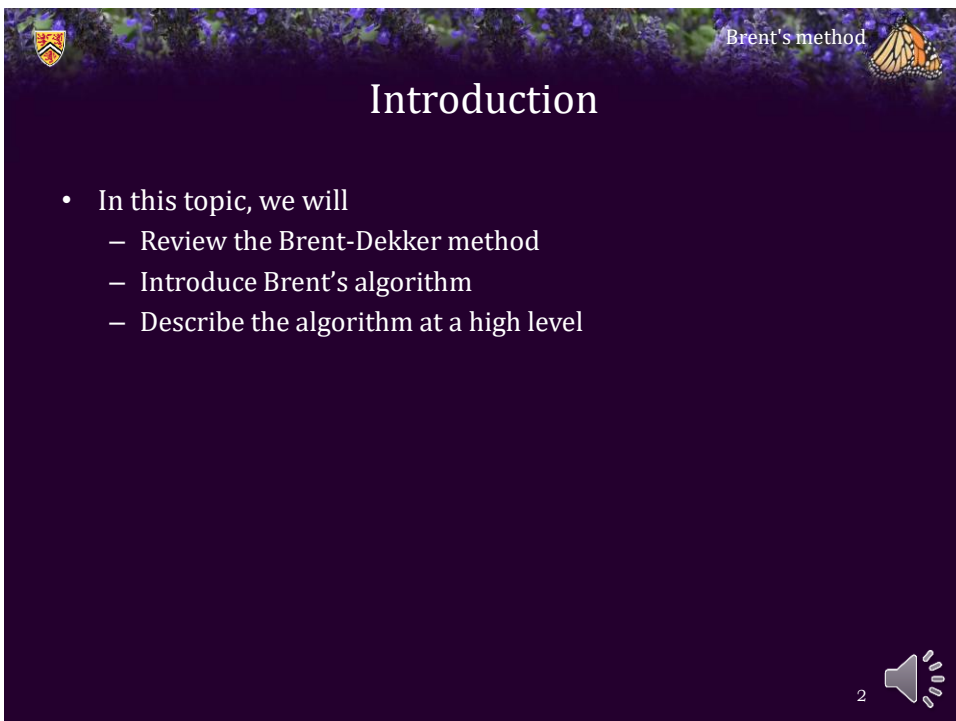
# Brent's method

Douglas Wilhelm Harder, LEL, M.Math.  
dwharder@waterloo.ca  
dwharder@gmail.com

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Brent's method



# Introduction

- In this topic, we will
  - Review the Brent-Dekker method
  - Introduce Brent's algorithm
  - Describe the algorithm at a high level

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Brent's method



## The Brent-Dekker method

- Recall the Brent-Dekker method:
  - A root-finding technique that alternates between:
    - The bisection method
    - The bracketed secant method
    - The inverse quadratic interpolation method
  - The first has the strongest guarantees of convergence
  - The last has the greatest rate of convergence
- The Brent-Dekker algorithm prefers the third technique where it appears to be converging, but can revert to one of the first two if the conditions seem to be unfavorable



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Brent's method

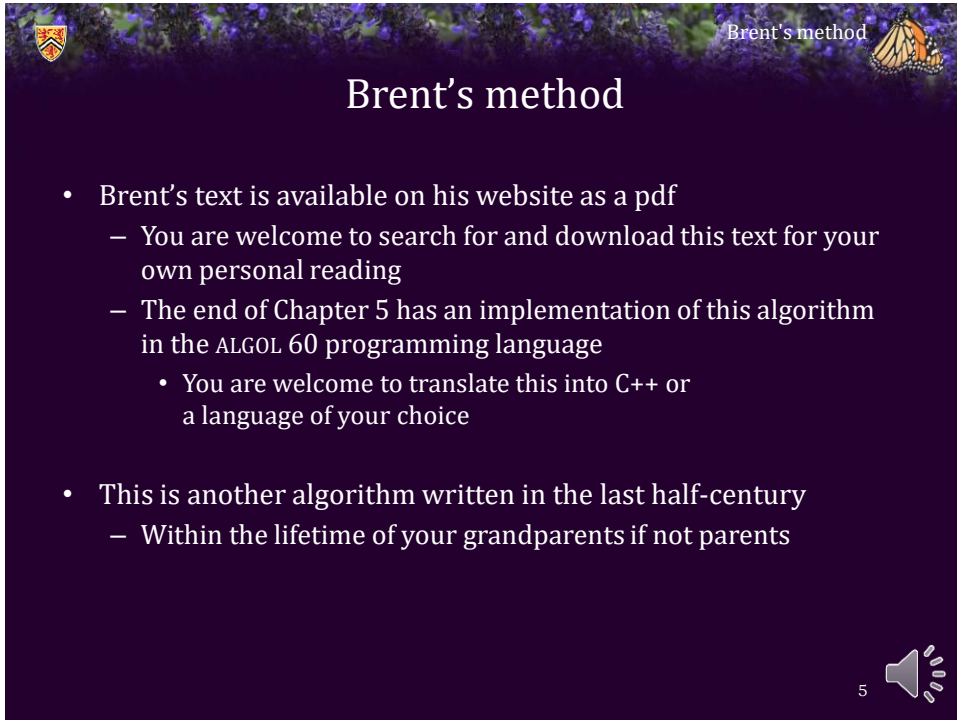
## Brent's method

- Brent, in the 1973 text  
 “Algorithms for minimization without derivatives”  
 details an algorithm that alternates between
  - The golden-ratio search
  - Successive parabolic interpolation
- The algorithm starts with the golden-ratio search
  - It then tries to use successive parabolic interpolation
  - Under certain conditions, it reverts to the golden-ratio search
    - For example, if after two steps of successive parabolic interpolation, the step size has not dropped by at least half
      - Recall with two steps of the golden-ratio search reduces the interval by  $\phi^{-2} \approx 0.381966$




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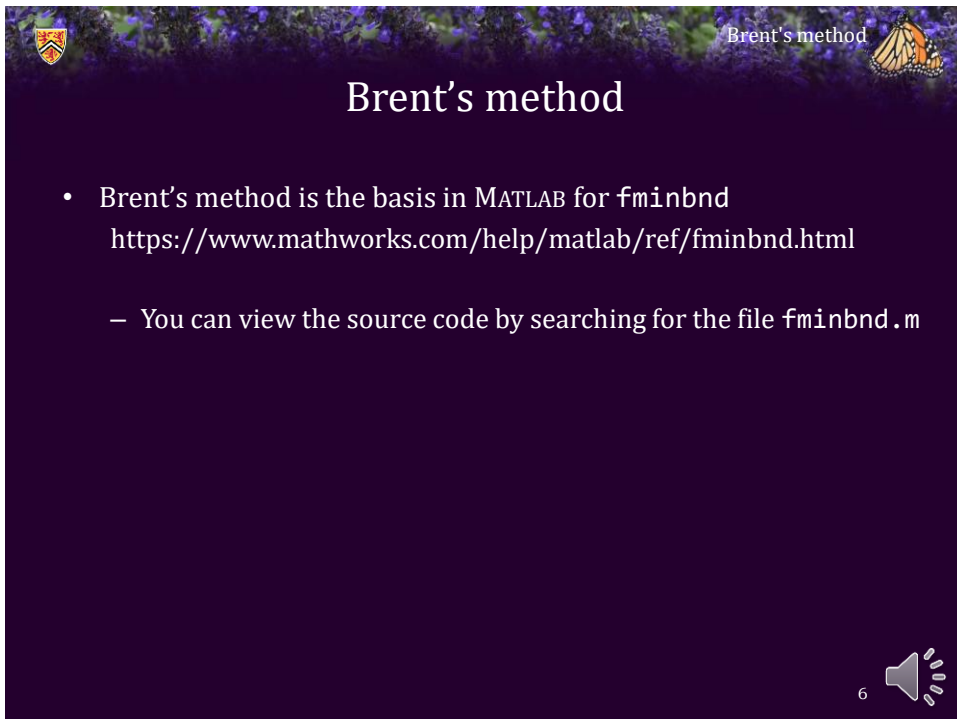


## Brent's method

- Brent's text is available on his website as a pdf
  - You are welcome to search for and download this text for your own personal reading
  - The end of Chapter 5 has an implementation of this algorithm in the ALGOL 60 programming language
    - You are welcome to translate this into C++ or a language of your choice
- This is another algorithm written in the last half-century
  - Within the lifetime of your grandparents if not parents


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## Brent's method

- Brent's method is the basis in MATLAB for `fminbnd`  
<https://www.mathworks.com/help/matlab/ref/fminbnd.html>
  - You can view the source code by searching for the file `fminbnd.m`

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Brent's method



## Summary

- Following this topic, you now
  - Are aware of Brent's method
  - Understand that algorithms like this exist and are available
  - Are aware that the text is available for reference



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Brent's method


## References


[1] Richard P. Brent, "Algorithms for minimization without derivatives", Prentice-Hall, 1973



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# Acknowledgments

None so far.



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# Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas. Mathematical equations are prepared in MathType by Design Science, Inc. Examples may be formulated and checked using Maple by Maplesoft, Inc.



The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see <https://www.rbg.ca/> for more information.






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